



ARTICLE V.

Determination of the Longitude of several Stations near the Northern Boundary of Ohio, from Transits of the Moon, and Moon-culminating Stars, observed in 1835, by Andrew Talcott, M. A. P. S., late Capt. U. S. Engineers. By Sears C. Walker, M. A. P. S. Read March 2, 1838.

SECTION I.

IN the summer of 1835, Captain Talcott was employed by the government of the United States to make a series of observations near the southern boundary of Michigan. The object of this mission was to settle the long disputed question of the Northern Boundary of Ohio, which, on the occasion of the proposed admission of Michigan into the Union, had been made the subject of a controversy, that threatened, for a while, to disturb the peaceful relations between the neighbouring states. Indeed, such was the pertinacity of the rival claimants, that an armed force was arrayed on each side, and a nice geographical question was on the point of being decided by a tribunal, of all others, least competent to do justice to its merits. The cause of this controversy, which fortunately terminated without fatal consequences, may no doubt be traced to an error in the map used by the parties to the original charter of Ohio. In this charter it was ordained that the northern boundary of Ohio should be the line running due east from the southernmost point of Lake Michigan, and terminating in the southernmost

point of North Cape, in the eastern extremity of Lake Erie. Subsequent observations by Captain Talcott have shown that North Cape is in latitude $41^{\circ} 44' 8''$, while the South Bend, so called, of Lake Michigan, is in $41^{\circ} 37' 6''$, leaving a discrepancy of about eight geographical miles. In 1817, deputy-surveyor Harris traced a boundary line from North Cape, Lake Erie, S. $87^{\circ} 42' W.$, towards South Bend, Lake Michigan. This was recognised by the citizens of Ohio as their true northern boundary, though differing from a parallel of latitude required by the other condition of the charter. On the contrary, another line fulfilling this condition of the charter was drawn by deputy-surveyor Fulton, in 1818, being a continuation eastward of the parallel of South Bend, Lake Michigan, to Lake Erie, seven miles south of the stipulated point, North Cape, and cutting off from Ohio the mouth of Maumee river, and the greater part of Maumee bay. This line was claimed as their true southern boundary by the citizens of Michigan, and hence the controversy referred to, for the facts of which I am indebted to Mr Henry S. Tanner.

To place this subject in its proper light before the interested parties, Captain Talcott was sent to make the necessary observations. The latitudes quoted above were obtained by a zenith micrometer, described in Pearson's *Astronomy*. The results furnished by this instrument present a remarkable uniformity. The probable error of a single observation does not exceed four hundred feet, and that of the mean for each station, excluding the errors of the star-catalogues, less than two hundred feet. It is hoped that these observations will be made public, as they will serve to make known in this country, one of the simplest and most accurate modes of determining latitudes by a portable instrument.

The labours of Captain Talcott were not confined to the determination of latitudes. At the two most important stations, South Bend, Lake Michigan, and Turtle Island, Lake Erie, as well as at Huron in Ohio, he observed a series of moon-culminations, the first, I believe, communicated to this society. They form an important and valuable contribution to the geography of Ohio, Indiana, Illinois and Michigan, and will be highly useful in perfecting the maps of those states. Indeed, these stations of Captain Talcott are the only well determined

points in the United States, north of the Ohio river; the longitudes of the maps of these northern states having hitherto been made to depend on the observations of Ellicott and De Ferrer, at points on the Ohio river, and on meridian lines traced from this river several hundred miles northward by the deputy-surveyors. When we consider the uncertainty of such operations, from the irregularity of local variations of the compass, it will appear somewhat remarkable that the maps of those states should have been so correct as they are shown to be from the result of Captain Talcott's observations. Thus we find for the position of Turtle Island, Lake Erie,

	<i>h.</i>	<i>m.</i>	<i>s.</i>
By Tanner's map U. S., 1837, Turtle Island west of Capitol, - - - - -	25	44	2
Deduct for error of Lambert's longitude of Capitol, used in this map, - - - - -		25	0
By the same map, corrected for Lambert's error, -	25	19	2
By Capt. Talcott's moon-culminations, Turtle Island, W. of Greenwich, - - - - -	5	33	31.8
Deduct longitude of the Capitol (Section XI., note), -	5	8	7.0
By Capt. Talcott's moon-culminations, T. Island west of Capitol, - - - - -	25	24	8
By Tanner's map, Turtle Island too far east, - -		5	6

A similar comparison for South Bend, Lake Michigan, gives,

South Bend, by Tanner's map, west of Capitol, - -	41	15	2
Deduct Lambert's error, - - - - -		25	0
Result by map, - - - - -	40	50	2
Result by moon-culminations, - - - - -	41	11	6
By Tanner's map, South Bend too far east, - -		21	4

SECTION II.

The full advantages afforded by moon-culminations, for perfecting geography, were first pointed out by Mr Nicolai, director of the Mannheim observatory, in 1821, in a paper which appeared in the first number of Schumacher's celebrated *Astronomische Nachrichten*. The same subject was shortly after proposed to the Astronomical Society of

London, by Francis Bailey, in a paper containing an original method for their reduction. This zealous astronomer selected lists of moon-culminating stars, and caused them to be distributed in advance to every observatory. The labours of Bessel, Hansen, Mollweide, Dumouchel, and others, have further developed the method of making and reducing this kind of observations. But the principal improvement was made in this method by the Nautical Almanac Committee, in causing the announcements of moon-culminations to be made in such a form, since 1834, that the reduction of them consists merely in the interpolation of the series there given for the right ascension of the moon's bright limb, at its upper and lower culminations at Greenwich.

These phenomena are now regularly observed at the British and continental observatories, and their longitude from each other has been determined by this method with an accuracy scarcely inferior to that of geodetic measures, powder signals, or the aggregate of observed occultations and eclipses. The observations for longitude, made by Captain Talcott, consist of one occultation, and three series of moon-culminations, at three different stations. In making the latter, a portable transit instrument was used, of three and a half feet focal length, and two inches and five-eighths aperture. Care was taken, previous to each moon-culmination, to adjust the horizontal axis by a delicate level, and the bias of the instrument was therefore as small as a temporary mounting would permit. The line of collimation of the instrument was adjusted for the mean of the wires, and does not appear to have undergone sensible change during the series, though frequent observations were made with the reversed axis, to detect the error, if any, in this adjustment. The deviation in azimuth was ascertained by observing high, low, and circumpolar stars, and a temporary meridian mark served to give steadiness to this adjustment. The sum of the deviations was usually less than 0.5 sec. in time. The results have been corrected for this sum, as far as it could be ascertained. It is difficult, with an instrument temporarily mounted, to furnish a greater degree of precision. The error arising from deviations so small is almost insensible, in the longitudes deduced from moon-culminations. The times were noted by calling out to assistants, and were registered on two chronometers by Brockbank, the assistant noting to the nearest

beat 0.4 sec. The probable error of a transit thus noted, for each of five wires (excluding the personal equation to which this and all other methods are liable), may be stated at 0.2 sec., or at most 0.3 sec. in time. Below are given the observations, corrected for deviations, and for rate and error of chronometers, in the usual form.

Station No. I. East of Huron, Ohio, Latitude 41°, nearly.

	Name.	Apparent R. A.			Wires.		Name.	Apparent R. A.			Wires.
		h.	m.	s.				h.	m.	s.	
1835, July 8,	Moon I.,	18	5	3.95	3	1835, July 10, 59	Sagittarii,	19	46	51.14	5
	φ Sagittarii,	18	35	23.14	3		Moon II.,	20	20	15.72	5
	σ Sagittarii,	18	45	4.20	3		υ Capricorni,	20	30	41.12	5
	ζ Aquilæ,	18	57	51.64	3		† Capricorni,	20	36	21.25	5
9,	Moon I.,	19	12	31.77	5	11,	Moon I.,	21	21	14.06	5
	Moon II.,	19	15	2.81	5		k Capricorni,	21	33	28.08	5
	γ Aquilæ,	19	38	27.02	5		δ Capricorni,	21	37	57.65	5
	59 Sagittarii,	19	46	51.09	5	12,	δ Capricorni,	21	37	57.46	2
10, h ²	Sagittarii,	19	26	42.17	5		Moon II.,	22	17	18.95	3

Station No. 2. Turtle Island, Lake Erie, Latitude 41° 45' 4".

	Name.	Apparent R. A.			Wires.		Name.	Apparent R. A.			Wires.
		h.	m.	s.				h.	m.	s.	
1835, Aug. 1, α	Virginis,	13	16	31.00	5	1835, Aug. 8,	Moon II.,	21	51	48.83	5
	α Bootis,	14	8	9.14	5		35 Aquarii,	21	59	58.11	5
	Moon I.,	14	35	39.61	5		σ Aquarii,	22	21	56.98	5
2,	Moon I.,	15	32	53.64	5	9,	35 Aquarii,	21	59	58.00	5
	α Serpentis,	15	36	9.95	5		σ Aquarii,	22	21	57.08	5
	δ Scorpii,	15	50	36.98	5		Moon II.,	22	45	36.43	5
	δ Ophiuchi,	15	5	43.83	5		α Piscis Aust.	22	48	33.81	5
3,	Moon I.,	16	33	43.62	5		α Pegasi,	22	56	35.16	5
	A Ophiuchi,	17	5	14.57	5		φ Aquarii,	23	5	48.94	3
	θ Ophiuchi,	17	11	55.16	5	10,	δ Aquarii,	22	45	55.74	5
6, μ	Sagittarii,	18	3	56.13	5		φ Aquarii,	23	5	48.64	5
	π Sagittarii,	18	59	59.26	5		Moon II.,	23	35	42.07	5
	Moon I.,	19	48	54.88	5		p Piscium,	23	50	15.65	5
	σ Capricorni,	20	9	54.52	5		r Piscium,	23	53	31.57	5
	π Capricorni,	20	17	54.75	5	11,	p Piscium,	23	50	15.65	5
7, σ	Capricorni,	20	9	54.43	5		r Piscium,	23	53	31.89	5
	π Capricorni,	20	17	54.51	5		Moon II.,	0	23	6.26	5
	Moon I.,	20	51	17.82	5		m Ceti,	0	44	36.59	5
	*Moon II.,	20	53	41.31	5	14,	Moon II.,	2	40	15.99	5
	χ Capricorni,	20	59	8.66	5		υ Tauri,	3	37	42.34	4
	ζ Capricorni,	21	17	16.72	5	16,	υ Tauri,	3	37	42.48	5
	β Aquarii,	21	22	54.36	5		Moon II.,	4	17	22.20	5
8, χ	Capricorni,	20	59	8.45	5		α Tauri,	4	26	28.33	5
	ζ Capricorni,	21	17	16.58	5						

* 0.17 sec. has been added to the time of the transit of the moon's II. limb, for defective illumination. See Table II.

Station No. 3. South Bend, Lake Michigan, Latitude 41° 37' 6".

<i>Name.</i>					<i>Apparent R. A. Wires.</i>					<i>Name.</i>					<i>Apparent R. A. Wires.</i>				
					<i>h.</i>	<i>m.</i>	<i>s.</i>								<i>h.</i>	<i>m.</i>	<i>s.</i>		
1835, Aug. 31,	Moon I.,	17	17	52.15	5					1835, Sep. 6,	μ Capricorni,	21	44	20.34	5				
	D Ophiuchi,	17	33	34.17	5						τ^2 Aquarii,	22	40	53.47	5				
	4 Sagittarii,	17	49	44.45	5						δ^2 Aquarii,	22	45	55.89	5				
Sept. 1,	4 Sagittarii,	17	49	44.41	5						\downarrow^3 Aquarii,	23	10	25.23	5				
	Moon I.,	18	22	10.07	5						*Moon I.,	23	13	2.95	5				
2,	ϕ Sagittarii,	18	35	23.03	5						Moon II.,	23	15	12.49	5				
	σ Sagittarii,	18	45	4.07	5						s Piscium,	23	56	55.53	5				
	Moon I.,	19	26	25.69	5						7, \downarrow^3 Aquarii,	23	10	24.86	5				
	c Sagittarii,	19	52	32.73	5						n Piscium,	23	39	30.21	5				
3,	59 Sagittarii,	19	46	51.44	5						s Piscium,	23	56	55.65	5				
	c Sagittarii,	19	52	32.88	5						Moon II.,	0	3	26.86	5				
	Moon I.,	20	28	35.30	5						8, s Piscium,	23	56	55.78	5				
	\downarrow Capricorni,	20	36	21.59	5						Moon II.,	0	49	54.54	3				
	n Capricorni,	20	55	2.97	5						c Piscium,	0	59	55.13	5				
4,	\downarrow Capricorni,	20	36	21.51	5						10, o Piscium,	1	36	43.16	5				
	n Capricorni,	20	55	2.87	5						ξ Piscium,	1	45	3.30	5				
	Moon I.,	21	27	14.72	5						Moon II.,	2	21	28.19	4				
6,	δ Capricorni,	21	37	58.11	5						μ Ceti,	2	36	3.87	5				
	μ Capricorni,	21	44	20.61	5						π Arietis,	2	40	7.29	4				

SECTION III.

To facilitate the final determination of the longitudes of these stations, a subsidiary Table I. has been prepared by interpolation from the series in the Nautical Almanac, which expresses the right ascension of the moon's bright limb at its upper and lower culmination at Greenwich. In this Table, employing the usual notation for series,

$\alpha_{0+t'}$ = The observed R. A., moon's bright limb, as given in list of moon-culminations for the several stations,

t' = The western longitude from Greenwich in seconds of time, which must be used as an argument, in order to interpolate from the series in the N. Almanac, the value of $\alpha_{0+t'}$ as observed,

$\log. n$ = Log. factor, to convert seconds of $\alpha_{0+t'}$ into seconds of t' ; in other words, to convert parts of the series into parts of the argument.

* 0.10 sec. have been subtracted from the time of transit of the first limb, for defective illumination. See Table II.

These values of t' and $\log. n$ have been computed, at my request, by E. O. Kendall.

TABLE I.

Date. 1835.		Limb.	No. of Sta- tion.	Observed Right Ascen- sion Moon's bright limb			Resulting Longitude from Greenwich not corrected for Tabular error			Log. n .
				$\alpha_0 + t'$			t'			
				<i>h.</i>	<i>m.</i>	<i>s.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>	
July	8	I	1	18	5	3,95	5	29	45,16	1.32818
	9	I	1	19	12	31,77			46,61	1.33355
	9	II	1	19	15	2,81			48,79	1.33384
	10	II	1	20	20	15,72			55,29	1.35684
	11	II	1	21	21	14,06			46,47	1.39095
	12	II	1	22	17	18,95			32,58	1.42817
Aug.	1	I	2	14	35	39,61	5	33	15,81	1.41364
	2	I	2	15	32	53,64			27,90	1.38756
	3	I	2	16	33	43,62			28,54	1.36166
	6	I	2	19	48	54,88			10,86	1.35128
	7	I	2	20	51	17,82			5,77	1.37718
	7	II	2	20	53	41,31			0,76	1.37720
	8	II	2	21	51	48,83			9,94	1.41091
	9	II	2	22	45	36,43			9,68	1.44408
	10	II	2	23	35	42,07			12,77	1.47204
	11	II	2	0	23	6,26			12,99	1.49155
	14	II	2	2	40	15,99			24,23	1.49046
	16	II	2	4	17	22,20			38,85	1.45126
	31	I	3	17	17	52,15	5	48	63,39	1.35460
	1	I	3	18	22	10,07			55,44	1.34786
	2	I	3	19	26	25,69			63,06	1.35545
Sept.	3	I	3	20	28	35,30			57,82	1.37610
	4	I	3	21	27	14,72			51,84	1.40489
	6	I	3	23	13	2,95			48,26	1.46575
	6	II	3	23	15	12,49			47,09	1.46590
	7	II	3	0	3	26,86			50,64	1.48462
	8	II	3	0	49	54,54			65,03	1.49654
	10	II	3	2	21	28,19			62,69	1.49294

The values of t' and n , in Table I., have been interpolated by an indirect process, which consists in deducing values of the argument corresponding to given values of the series. A direct method of computing the unknown quantity t' has been given by Mollweide, *Astr. Nachr.*, No. 40. I have preferred the formulæ given by Bessel, No. 33, of the same. As his modification of Newton's formula for inter-

polation adapted to this subject does not appear to have been translated into English, I propose to give the substance of it here. The grounds of preference of it are, its rapid convergency, and its adaptation to computation by logarithms.

For the arguments,

$$-2, -1, -0, +1, +2, +3,$$

let the values of a function be

$$a_{,,,}; a_{,,}; a_{,}; a'; a''; a'''.$$

Denoting the differences as in the following scheme,

:	I	II	III	IV	V
a	:				
	b	c			
a	b		d		
a	b	c		e	
a	b		d		f
a	b	c		e	
a			d		
a	b	c			
a	:				
:					

and making

$$a = \frac{1}{2} (a_{,} + a')$$

$$c = \frac{1}{2} (c_{,} + c')$$

$$e = \frac{1}{2} (e_{,} + e')$$

$$g = \frac{1}{2} (g_{,} + g')$$

then the value of the function for any argument t , expressed in parts of the constant interval unity,

$$= a + \frac{t - \frac{1}{2}}{1} b + \frac{t \cdot t - 1}{1 \cdot 2} c + \frac{t \cdot t - 1 \cdot t - \frac{1}{2}}{1 \cdot 2 \cdot 3} d$$

$$\begin{aligned}
 & + \frac{t + 1.t.t - 1.t - 2}{1.2.3.4} e + \frac{t + 1.t.t - 1.t - 2.t - \frac{1}{2}}{1.2.3.4.5} f \\
 (1) \quad & + \frac{t + 2.t + 1.t.t - 1.t - 2.t - 3}{1.2.3.4.5.6} g
 \end{aligned}$$

Also the variation of the function for the unit of interval at the rate for the arrangement t , being the first differential quotient of the above formula,

$$\begin{aligned}
 & = \frac{b}{1} + \frac{2t - 1}{1.2} c + \frac{3t^2 - 3t + \frac{1}{2}}{1.2.3} d \\
 (2) \quad & + \frac{4t^3 - 6t^2 - 2t + 2}{1.2.3.4} e + \frac{5t^4 - 10t^3 + 5t - 1}{1.2.3.4.5} f \\
 & + \frac{6t^5 - 15t^4 - 20t^3 + 45t^2 + 8t - 12}{1.2.3.4.5.6} g^*
 \end{aligned}$$

If we denote, for conciseness, the co-efficients of b, c, d , &c., in (1), by $X, X', X'',$ &c.; and those of e, d, e , &c., in (2), by $T, T', T'',$ &c., we shall have :

Value of function for argument t , $= a + bX + cX' + dX'' + eX'''$, &c.

Rate of variation for argument t , $= b + cT + dT' + eT'' + fT'''$, &c.

To apply these formulæ to the reduction of moon-culminations, for an assumed meridian t seconds in time + west of Greenwich, we must make the argument the difference of meridians, the unit of interval being 43200 seconds. Two sets of coefficients are required for each

* The coefficient of g in (2), in Bessel's paper, is thus stated, owing to a typographical error.

$$\frac{6t^5 - 15t^4 - 40t^3 + 90t^2 + 18t - 27}{1.2.3.4.5.6}$$

new argument. The values of $T, T', \&c.$ for the even hours, or twelfth parts of the argument, are given by Bessel; those of $X, X', \&c.$ by Nicolai, *Astr. Nach.*, 37. Where a single, or few observations are required to be reduced for a particular station, the labour of computing $T, T', \&c.$ may be saved by means of the following transformation, which I do not recollect to have seen in any publication.

The comparison of (1) and (2) gives,

$$cT = c.X$$

$$dT' = d \left\{ X' + \frac{1}{3.4} \right\}$$

$$eT'' = e \left\{ X'' - \frac{1}{2.3} X \right\}$$

$$fT''' = f \left\{ X''' + \frac{1}{3.4} X' - \frac{1}{4.5.6} \right\}$$

$$gT^{iv} = g \left\{ X^{iv} - \frac{1}{2.3} X'' + \frac{1}{2.3.5} X \right\}$$

$$hT^v = h \left\{ X^v + \frac{1}{3.4} X''' - \frac{1}{4.5.6} X' + \frac{1}{4.5.6.7} \right\}$$

$$iT^{vi} = i \left\{ X^{vi} - \frac{1}{2.3} X^{iv} + \frac{1}{2.3.5} X'' - \frac{1}{4.5.7} X \right\}$$

whence, making

$$\alpha = b + \frac{1}{3.4} d - \frac{1}{4.5.6} f + \frac{1}{4.5.6.7} h -, \&c.$$

$$\beta = c - \frac{1}{2.3} e + \frac{1}{2.3.5} g - \frac{1}{4.5.7} i +, \&c.$$

$$\gamma = d + \frac{1}{3.4} f - \frac{1}{4.5.6} h +, \&c.$$

$$\delta = e - \frac{1}{2.3} g + \frac{1}{2.3.5} i -, \text{ \&c.}$$

$$\varepsilon = f + \frac{1}{3.4} h -, \text{ \&c.}$$

$$\zeta = g - \frac{1}{2.3} i +, \text{ \&c.}$$

and calling

V = the hourly variation for argument t , we have, enclosing in brackets the log. co-efficient,

$$V = \frac{1}{12} (\alpha + \beta X + \gamma X' + \delta X'' + \varepsilon X''' + \zeta X^{vi} +, \text{ \&c.})$$

$$n = \frac{3600}{V} = [3.55630] \frac{1}{V}$$

The value of V is the same as that given in the N. Almanac, viz. the variation of the R. A. of the moon's bright limb in one hour of longitude, and may be obtained for the argument t , as above, from the values of V, for Greenwich.

Although I have given several terms in the series for the value of V, three are, in all instances, sufficient for reducing moon-culminations; hence, adopting the usual notation for the argument of a series, the formulæ used in computing Table I. may be briefly recapitulated, t being an approximate longitude, differing less than a minute from the true longitude,

$$\alpha_{0+t} = a + bX + cX' + dX'' + eX'''$$

$$n = n_{0+t} = [4.63548] \frac{1}{\alpha + \beta X + \gamma X'}$$

$$t' = t + n (\alpha_{0+t'} - \alpha_{0+t} + \frac{1}{n+1} \cdot \theta)$$

where,

$$\theta = \text{the correction of the sidereal time of observation}$$

VI.—3 N

$$\begin{aligned}
X &= [5.3645163] \cdot (t - 6^h 0^m 0^s) \\
X' &= [0.42800] t \cdot (t - 12^h 0^m 0^s) \\
X'' &= [9.5229] X X' \\
X''' &= [9.6499] X' \cdot (t + 12^h 0^m 0^s) (t - 24^h 0^m 0^s) \\
*X^{iv} &= [9.3010] X X'''.
\end{aligned}$$

SECTION VI.

The observed increase of the right ascension of the moon's bright limb, as derived directly from the lists of moon-culminations, requires a correction, when the same stars have not been observed at both places, as well as when the number of wires used at each place is not uniform. The formula for computing this correction has been derived from a combination of Gauss's application of the calculus of probabilities to the reduction of moon-culminations, as given by Nicolai, *Astr. Nachr.*, No. 26, with Dumouchel's method, No. 125 of the same, for the different stars. Thus, for the European observatory and western station respectively, let

$$\begin{aligned}
A' \text{ and } A &= \text{the observed R. A. of a star,} \\
E &= A' - A \text{ for the same star,} \\
E &= \text{a similar value for another star,} \\
l \text{ and } l' &= \text{the number of wires on which each limb was observed,} \\
a \text{ and } a' &= \text{similar values for a star,} \\
\lambda &= \frac{ll'}{l+l'}, \text{ for the moon's limb,} \\
\mu &= \frac{aa'}{a+a'}, \text{ for one star,} \\
\mu' &= \text{a similar value for another star,} \\
\varepsilon &= \text{the correction of the observed increase of the right ascension of the moon's bright limb,} \\
\Sigma &= \text{symbol to denote the aggregate of similar quantities.}
\end{aligned}$$

Then we have,

* This term is not required for reducing moon-culminations.

$$\varepsilon = \frac{\sum (E \cdot \frac{\lambda \mu}{\lambda + \mu})}{\sum \frac{\lambda \mu}{\lambda + \mu}}$$

SECTION VII.

In order to deduce the final correction of t' in Table I., let τ , τ' , ν and θ' denote for the European observatory values corresponding respectively to t , t' , n and θ , for a western station. Effecting the interpolation by means of the same values of a , b , c , &c.; α , β , γ , &c.; and with constant values of X , X' , &c., for the known value of T , or longitude of each observatory from Greenwich, we derive for the longitude of the western station, independent of the stars' and moon's right ascensions in the N. Almanac,

$$T = t + \nu \left(\alpha_{0+\tau} - \alpha_{0+\tau'} + \varepsilon - \frac{\theta'}{\nu + 1} \right)$$

Also calling W the *weight* of each result, and η the probable error of a transit over a single wire (assumed equal ± 0.2 sec.), and making $\sigma = \mu + \mu' + \mu'' + \dots$, we have, after Gauss's method, quoted above,

$$W = \frac{\lambda \sigma}{(\lambda + \sigma) n n}$$

$$\text{Probable error of final result for each station} = \frac{\eta}{\sqrt{\sum \frac{\lambda \sigma}{(\lambda + \sigma) n n}}}$$

The values of T , for the several Stations, are given below. The weights of the Cambridge and Edinburgh observations are computed on the supposition that the moon and stars are observed on five wires, as this number is not stated in the lists of moon culminations published in Mem. Royal Astron. Soc. When both limbs are observed at the same culmination, l or l' is the sum of the wires for both limbs. The result of a single comparison of a western station has the weight w . The result for each day has the weight W computed by the above formula, making a' and l' equal to the sum of all the wires on which the moon or a star was observed at all the European observatories.

In a few instances an observation at the western station is compared with a European observation of the succeeding day, in which case $T = 24^h$ — western longitude of Greenwich from the station.

Station No. 1. East of Huron, Ohio.

Date. 1835.	Limb.	Observatory com- pared with	Date.	Limb.	Resulting Longitude.	<i>w</i> for each com- parison.	Mean result for each day.	W for each day.
July	8	I	8	I	<i>h. m. s.</i> 5 29 48.4	0.00276	<i>h. m. s.</i> 5 29 46.65	0.00339
	8	I		I	45.2	0.00331		
	9	I & II	9	I	53.9	0.00307	53.90	0.00307
	10	II	10	II	56.4	0.00387	56.40	0.00387
Final Result, Station No. 1, West of Greenwich, Probable Error,							<i>h. m. s.</i> 5 29 52.46 ± 2.95	0.01033

Station No. 2. Turtle Island, Lake Erie.

Date. 1835.	Limb.	Observatory com- pared with	Date.	Limb.	Resulting Longitude.	<i>w</i> for each com- parison.	Mean Result for each day.	W for each day.
Aug.	3	I	3	I	<i>h. m. s.</i> 5 33 34.5	0.00315	<i>h. m. s.</i> 5 33 32.62	0.00384
		I	3	I	30.1	0.00236		
6	I	Edinburgh	6	I	34.4	0.00248	26.99	0.00488
	I	Cracow	6	I	20.5	0.00324		
	I	Dorpat	6	I	27.9	0.00296		
7	I & II	Edinburgh	7	I	32.1	0.00406	29.92	0.00680
	I & II	Greenwich	7	I	31.5	0.00406		
	I & II	Cambridge	7	I	25.6	0.00352		
8	II	Greenwich	8	II	39.0	0.00301	33.72	0.00402
	II	Cambridge	8	II	28.1	0.00283		
9	II	Greenwich	9	II	37.6	0.00237	34.73	0.00381
	II	Cambridge	9	II	29.3	0.00216		
	II	Dorpat	9	II	36.9	0.00226		
10	II	Greenwich	10	II	40.3	0.00190	40.30	0.00190
11	II	Greenwich	11	II	36.2	0.00173	31.24	0.00232
	II	Dorpat	11	II	25.6	0.00152		
Final Result, Turtle Island, West of Greenwich, Probable error,							<i>h. m. s.</i> 5 33 31.82 ± 1.81	0.02757

Station No. 3. South Bend, Lake Michigan.

Date. 1835.	Limb.	Observatory compared with	Date.	Limb.	Resulting Longitude.	W for each com- parison.	Mean Result for each day.	W for each day.
					<i>h. m. s.</i>		<i>h. m. s.</i>	
Aug. 31	I	Edinburgh	31	I	5 49 27.3	0.00326		
31	I	Cambridge	31	I	21.3	0.00244	5 49 24.73	0.00414
Sept. 1	I	Cambridge	1	I	19.4	0.00288	19.40	0.00288
2	I	Greenwich	2	I	12.6	0.00324		
2	I	Cambridge	2	I	20.5	0.00324		
2	I	Cracow	2	I	10.4	0.00365		
							14.33	0.00608
3	I	Kremsmunster	3	I	16.7	0.00332		
3	I	Dorpat	3	I	18.5	0.00307		
							17.56	0.00459
4	I	Edinburgh	5	I	25.6	0.00194		
4	I	Greenwich	5	I	14.0	0.00194		
4	I	Cracow	5	I	8.5	0.00346		
4	I	Dorpat	4	I	14.3	0.00294		
							14.42	0.00432
6	I & II	Edinburgh	6	I & II	23.1	0.00351		
6	I & II	Cambridge	6	I & II	16.6	0.00293		
							20.14	0.00355
7	II	Cambridge	7	II	14.8	0.00135		
7	II	Dorpat	7	II	23.2	0.00157		
							19.32	0.00245
8	II	Greenwich	8	II	25.6	0.00127		
8	II	Dorpat	8	II	26.0	0.00134		
							25.81	0.00173
Final Result, South Bend West of Greenwich,							<i>h. m. s.</i>	
Probable error,							5 49 18.55	0.02974
							\pm 1.74	

SECTION IX.

The longitudes of these stations may perhaps require a further correction for the comparative *irradiation* of Captain Talcott's and the European transit instruments. This subject, though frequently discussed, is still left in uncertainty. Corresponding observations, with telescopes of different optical capacity, indicate that the apparent diameter of the moon is subject to a small variation, depending upon this capacity, and upon the degree of illumination of the wires. If this were the only effect of irradiation, it could be easily allowed for by reducing

the diameter, in the Nautical Almanac, to the same dimensions as that which has been observed. It is obvious that the error arising from this source must vanish, when each limb of the moon has been observed the same number of times, and with equal weights. Though this can hardly be expected, still, the error would vanish on using the mean of a great number of results for each limb, and giving equal weights to the results by each limb. The mode of deducing the correction of this error is given below, for the several instruments, and is derived from the observed interval between the transits of the two limbs of the moon, when nearly full; this duration being corrected for the defective illumination of one of the limbs. It appears from experience that there still remains an error of irradiation, which no multiplication of observations by the same observers, with fixed telescopes, can completely remove. Thus the Dorpat and Paris transit instruments appear to be liable to a constant error of this kind; and the difference of longitude between those observatories, derived from moon-culminations, cannot, without correcting for it, be made to agree with the results of occultations and of geodetic measurements. Argelander found that his transit instrument at the Abo observatory, while it gave correct longitudes, when compared with several instruments of nearly equal capacity at the German observatories, required a constant correction to reconcile with these the results by the Greenwich ten feet transit instrument. Again, Dr Robinson finds that without the application of such a correction, it is impossible to deduce a correct difference of longitude by the Greenwich and Armagh transit instruments. In some of the instances referred to, the outstanding error, even when the mean of the results by both limbs is used, amounts to three seconds of longitude in time. Dr Robinson has proposed to deduce this correction by means of comparison of the observed diameter of the sun, as deduced from the transits by the same instruments. Though successful, in his own case, I do not know that his method has been generally adopted. I will here make a remark which I have not noticed in any papers on this subject, that it seems to me highly probable that there is a personal equation, arising from the difficulty of noticing the precise instant when the moon's limb is tangent to the centre of the wire of a transit instrument. If such be the

case, it must vary with the observer, and with the optical capacity of the instrument used. It would also vary with the different limbs; since the transit of the first limb exhibits the approach to tangency of the convex side of an arc, that of the second limb of the concave side of the same. Granting the existence of such an equation, it could hardly be the same for each limb; the difference, then, remains constant, with the same observer and same instrument, and cannot be eliminated otherwise than by a multiplication of observers and instruments. Whatever be the cause of the error of irradiation, experience has shown that the results of moon-culminations, like those of eclipses of Jupiter's satellites, approach nearer to the truth in proportion as the instruments approach to equality in their optical powers. The method of computing the correction of Burckhardt's semidiameter, from observations of both limbs of the moon, when nearly full, is given by Professor Airy, in the Greenwich Observations for 1836. His method, combined with Encke's formulæ, in the Berlin Jahrbuch for 1832, p. 251, may be thus analytically expressed.

- S = the correct sidereal time of the moon's semidiameter passing the meridian,
 S' = the computed time,
 $2 I$ = the observed duration of the transit of the moon's defective diameter,
 α = the sid. time of U. C. of moon's defective limb,
 A and D = the sun's R. A. and dec.,
 $(S - I)$ = $S' \cos. D (1 + \cos. (\alpha - A))$
 = compliment of duration of transit of moon's defective diameter,
 i = $S - S' =$ correction of Burckhardt's semidiameter,
 m = the increase of the R. A. of the moon's bright limb in arc, in a lunar day,
 κ = Burckhardt's constant value of $\frac{\text{moon's semidiameter,}}{\text{moon's horizontal parallax,}}$
 π = the moon's horizontal equatorial parallax,
 δ = the moon's true declination,
 $S' = \frac{360^\circ + m}{360^\circ} \cdot \kappa \pi \sec. \delta$

The value of i for the Greenwich transit instrument, viz. $+ 0.2$ sec. in time, or $+ 3''$ in space, is given by the Astronomer Royal, and is found to agree precisely with that which he derives from the mural circles, from similar observations of the vertical diameter of the moon, corrected for the defective illumination. This coincidence would seem to show that this correction is required by the actual dimensions of the moon, and that if other transit instruments indicate a different correction, it must be from inferior optical capacity.

For the other instruments, with which Captain Talcott's must be compared, I have computed, by the above formulæ, the requisite correction of Burckhardt's semidiameter, as far as it could be derived from all the observations of the transit of both limbs of the moon, on the same day, which I have been able to find. The correction for the mean of all the results is $+ 0.15$ sec. in time, or $2''.25$, in space. The separate results are given in the following Table.

TABLE II.

Date.	Observatory.	Moon's Limb.	Sid. time U. C.	Half com- pliment of Defective Diameter S — I.	S	S'	$i = S - S'$	Mean i .
1835, July 9	Station No. 1.	I	<i>h. m. s.</i> 19 12 31.77	<i>s.</i> + 0.00	<i>s.</i> 75.52	<i>s.</i> 75.47	<i>s.</i> + 0.05	
Aug. 7	Turtle Island	II	19 15 2.81					
	Lake Erie	I	20 51 17.82	+ 0.08	71.74	71.85	— 0.11	
" Sept. 6	South Bend	II	20 53 41.14					<i>s.</i>
	Lake Michigan	I	23 13 3.05	+ 0.05	64.77	64.79	— 0.02	— 0.03
" " 6	Edinburgh	II	23 15 12.49					
		I	23 1 23.18	+ 0.00	65.17	65.01	+ 0.16	+ 0.16
		II	23 3 33.52					
1836, Jan. 3	Greenwich	I	6 54 0.46	+ 0.00	69.03	68.84	+ 0.19	
		II	6 56 18.52					
" July 27	"	I	20 17 44.10	+ 0.06	76.60	76.35	+ 0.25	
		II	20 20 17.19					
" Sept. 24	"	I	0 10 37.86	+ 0.02	65.66	65.46	+ 0.20	+ 0.20
		II	0 12 49.15					
1831, July 24	Cambridge	I	20 14 28.55	+ 0.00	65.97	65.73	+ 0.24	
		II	20 16 40.48					
1832, Feb. 15	"	I	9 46 34.31	+ 0.01	70.43	70.30	+ 0.13	
		II	9 48 55.14					
" Aug. 11	"	I	21 42 44.20	+ 0.08	63.89	63.63	+ 0.26	
		II	21 44 51.83					
1833, May 3	"	I	14 46 11.42	+ 0.00	67.38	67.13	+ 0.25	
		II	14 48 26.18					
" Oct. 28	"	I	2 32 38.82	+ 0.12	63.84	63.72	+ 0.12	
		II	2 34 46.26					
1834, May 22	"	I	15 58 44.84	+ 0.00	71.66	71.40	+ 0.26	
		II	16 1 8.15					
" Sept. 17	"	I	23 47 44.27	+ 0.02	61.28	61.32	— 0.04	
		II	23 49 46.79					
1835, Sept. 6	"	I	23 0 55.70	+ 0.00	65.33	65.03	+ 0.30	+ 0.20
		II	23 3 6.36					
1832, July 12	Kremsmunster	I	19 27 2.83	+ 0.00	65.20	65.11	+ 0.09	
		II	19 29 13.23					
1834, Sept. 17	"	I	23 46 30.18	+ 0.01	61.50	61.36	+ 0.14	+ 0.12
		II	23 48 33.17					
" Feb. 23	Cracow	I	10 37 16.94	+ 0.06	71.41	71.47	— 0.06	
		II	10 39 39.64					
" May 22	"	I	15 56 25.20	+ 0.00	71.55	71.34	+ 0.21	
		II	15 58 48.30					
1835, June 10	"	I	17 11 19.04	+ 0.00	76.07	75.97	+ 0.10	+ 0.08
		II	17 13 51.18					
1833, Oct. 28	Dorpat	I	2 29 7.64	+ 0.09	63.92	63.60	+ 0.32	
		II	2 31 15.40					
1835, Oct. 6	"	I	1 1 19.41	+ 0.06	62.06	61.84	+ 0.22	+ 0.27
		II	1 3 23.42					

Calling

i = correction of Burckhardt's semidiameter for Captain Talcott's transit instrument,

i' = a similar correction for a European instrument,

ι = $\mp (i' - i)$ = comparative correction,

the above table gives,

$\iota^{(0)}$	=	\mp	0.19	for Captain Talcott's instrument with the Edinburgh,
$\iota^{(1)}$	=	\mp	0.23	" " Greenwich,
$\iota^{(2)}$	=	\mp	0.23	" " Cambridge,
$\iota^{(3)}$	=	\mp	0.15	" " Kremsmunst.,
$\iota^{(4)}$	=	\mp	0.11	" " Cracow,
$\iota^{(5)}$	=	\mp	0.30	" " Dorpat.

Then T , being the result of a single comparison of Captain Talcott's instrument with the Edinburgh, &c., we have the several results corrected for the comparative values of the correction of Burckhardt's semidiameter, the upper sign for the first limb, the lower for the second,

$$\begin{aligned}
 &= T \mp n \iota^{(0)} \\
 &= T \mp n \iota^{(1)} \\
 &= \&c.
 \end{aligned}$$

Applying the correction for all the instruments in this manner, and taking the means for each day according to the weights w , and the means for the several days according to the weights W , we find the correction for this part of irradiation, of the final result for each station, to be:

Correction for station No. 1, near Huron, Ohio,	—	0.72 sec.
" " 2, Turtle Island,	+	0.28 sec.
" " 3, South Bend,	—	2.56 sec.

This hypothetical correction, it appears, must be rejected; for on submitting it to the best test which the nature of the subject furnishes,

the sum of the squares of the errors of the single results derived from its application, far exceeds that which arises from the neglect of it.

SECTION X.

Reduction of Captain Talcott's observation of the occultation of τ^2 Aquarii, at Station No. 2, Turtle Island, Lake Erie, August 9, 1835. Latitude $41^\circ 45' 9''$, longitude, as above, $5^h 33^m 31.82$ sec.

	By Brockbank's gold chronometer.			Sidereal Time.			Mean Time.		
	<i>h.</i>	<i>m.</i>	<i>s.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>	<i>h.</i>	<i>m.</i>	<i>s.</i>
Immersion τ^2 Aquarii, *	20	7	59.40	20	8	35.10	10	56	49.55
Emersion " "	20	45	1.60	20	45	37.37	11	33	46.15

Using Bessel's method (*Beiträge zur Theorie der Finsternisse, u. s. w.*) Astr. Nachr. No. 152, and enclosing in a parenthesis the letters of his notation, and substituting for his d and d' their equivalents in the notation above, viz. — T and — t' , we derive, from the moon's and stars' place in the Nautical Almanac,

$$(T) = 17^h 0^m 0^s = \text{mean time Greenwich.}$$

$$(P) = -0.195736$$

$$(Q) = +0.624923$$

$$(N) = +67^\circ 46' 55.8'' + (T') \times 8.1''$$

$$\log \cos(\delta) = +9.987162$$

$$\log\left(\frac{S}{n}\right) = +3.793159 + (T') \times 0.000015$$

$$- t' = -5^h 33^m 19.57^s + 1.814 \times (\epsilon) + 4.357 \times (\zeta) + 4.720 \times (\eta).$$

$$- t' = -5 32 50.53 + 1.814 \times (\epsilon) - 2.807 \times (\zeta) - 3.343 \times (\eta).$$

The Greenwich observations, on the 9th and 10th, give, for the corrections of the moon's place, at the time of the occultation,

$$15 \times \Delta(\alpha) = - 16.38''$$

$$\Delta(\delta) = - 2.30''$$

* The time of beginning has been increased one minute, a correction found necessary, in several instances, in reducing the transit observations.

The Dorpat observations of the moon-culmination that night, give

$$\begin{aligned} 15 \times \Delta (\alpha) &= - 14.71'' \\ \Delta (\delta) &= \text{not stated.} \end{aligned}$$

The mean corrections are,

$$\begin{aligned} 15 \times \Delta (\alpha) &= - 15.54'' \\ \Delta (\delta) &= - 2.30'' \end{aligned}$$

Also by Bessel's formulæ,

$$\begin{aligned} (\epsilon) &= \sin (N) \cos (\delta) \Delta \alpha + \cos (N) \Delta (\delta) = - 14.83'' \\ (\zeta) &= - \cos (N) \cos (\delta) \Delta \alpha + \sin (N) \Delta (\delta) = + 3.57'' \end{aligned}$$

whence,

$$\begin{aligned} \text{by imm.} - T &= - 5^h 33^m 30.7^s + 4.720 \times (\eta) \\ \text{by emer.} - T &= - 5 33 27.6 - 3.343 \times (\eta) \\ \text{mean,} &= - 5 33 29.2 + 0.688 \times (\eta) \end{aligned}$$

A result which agrees with the mean of the longitudes by moon-culminations, viz., $5^h 33^m 31.8^s$, more nearly than could have been expected, when we consider the largeness of the co-efficients ϵ , ζ and η . These results are derived from the assumption that Burckhardt's semidiameter needs no correction, in which case (η) would be equal to 0. If, however, we adopt Airy's correction for the results by meridian observations $+ 0.2$ sec. in time, whence $\eta = + 3''$, and apply this correction to the results above, viz. to those for $\Delta \alpha$ and $\Delta \delta$, as well as to η , we derive,

$$\begin{aligned} \text{by immer.} - T &= - 5^h 33^m 31.0 \\ \text{by emer.} - T &= - 5 33 39.7 \\ \text{mean,} - T &= - 5 33 35.3 \end{aligned}$$

It does not appear, from experience, that Burckhardt's semidiameter requires an additive correction, for occultations of small stars; on the contrary, most computers apply a negative correction of $- 2.5''$ to the value of η : this applied to the former mean result, would give,

$$\begin{aligned} T &= 5^h 33^m 30.9, \text{ by the occultation;} \\ \text{also, as above, } T &= 5 33 31.8 \text{ by all the moon-culminations.} \end{aligned}$$

SECTION XI.

ON THE LONGITUDE OF THE CAPITOL AT WASHINGTON.

Having in the early part of this memoir alluded to the error of Lambert's value for the longitude of the Capitol, I shall here cite the authorities on which such a statement is founded. The American Almanac has, for years, pointed out this error in general terms, without however tracing it to its source, viz., the omission, on the part of Lambert, to correct his results by corresponding observations, for the errors of Burg's Tables, used in computing the Nautical Almanac. All the observations yet published at Washington, or its immediate vicinity, from which its longitude can be computed, are seven in number. The results derived from them, with the names of the observers and computers, are contained in the following table.

Phenomenon observed.	Longitude W. of Greenwich.			Remarks.
Solar eclipse of April 3, 1791, observed at Georgetown, D. C., by Andrew Ellicott.	<i>h.</i>	<i>m.</i>	<i>s.</i>	
	5	8	4.3	By Bowditch.
			6.2	By De Ferrer.
Occultation of Aldebaran, Jan. 21, 1793, by Andrew Ellicott, supposed to have been observed on the site of the Capitol.	5	7	51.6	By Triesnecker.
			54.4	By Wurm.
			6.4	By Lambert.
				With corresponding observations at Greenwich and Paris.— <i>Mem. A. A. S.</i> , vol. III., p. 269.
				With same corr. obs.— <i>Mem. A. P. S.</i> , vol. VI., p. 359.
				With meridian observations at Greenwich and Paris.— <i>Ephem. Vindob.</i> , 1806.
				With meridian observations near Thoulouse.— <i>Astr. Nachr.</i> No. 21.
				<i>Mem. A. P. S.</i> , N. Series, vol. I., p. 106. <i>This result must be rejected, because affected with the errors of Burg's Tables, which are eliminated as above by Triesnecker and Wurm.</i>

Phenomenon observed.	Longitude W. of Greenwich.			Remarks.
Occultation of Alcyone, Oct. 20, 1824, observed near the Capitol, by Seth Pease.	<i>h.</i>	<i>m.</i>	<i>s.</i>	With corresponding observations at Vienna, Dessau, and Hohenneiche.— <i>Astr. Nachr.</i> , 91. Ib., p. 109.— <i>Rejected, for reason similar to above.</i>
	5	8	6.8 By Wurm.	
		7	39.8 By Lambert.	
Solar eclipse of Sept. 17, 1811, observed near the Capitol, by Seth Pease.	5	8	11.4 By Bowditch.	With corresponding observations at Salem, Massachusetts.—Ib., p. 269. With corresponding observations at Salem, N. Haven, and Bowdoin College, and N. York.— <i>Astr. Nachr. No. 181.</i> Ib., p. 114.— <i>Rejected, as above.</i>
			6.2 By Wurm.	
			21.6 By Lambert.	
Occultation of γ Tauri, Jan. 12, 1813, observed near the Capitol by Seth Pease.	5	8	27.3 By Wurm.	With corresponding observation near Marseilles.— <i>Ast. Nach.</i> , No. 21. This observation not good—local time not well determined. Ib., p. 114.— <i>Rejected for double reason.</i>
	5	7	45.5 By Lambert.	
Solar eclipse of Feb. 12, 1831, observed by F. R. Hassler.	5	8	7.2 By Paine.	With corresponding observations at W. C. Bond's observatory, Dorchester, Mass., and at Monomoy Point.— <i>Am. Almanac.</i>
Solar eclipse of May 15, 1836, observed by F. R. Hassler.	5	8	13.5 By S. C. Walker.	With corresponding observations at the principal observatories in Europe, reduced by H. C. F. Peters.— <i>Astr. Nachr.</i> , No. 326.

Taking the mean of the results obtained by the different computers, (those of Lambert being rejected, for reasons mentioned above) we have, for the longitude of the Capitol,

	<i>h.</i>	<i>m.</i>	<i>s.</i>
(1)	5	8	5.3
(2)		7	53.0
(3)		8	6.8

	<i>h.</i>	<i>m.</i>	<i>s.</i>
(4)	8	8	8.7
(5)	8	27	4
(6)	8	7	2
(7)	8	13	5
Mean of seven results	5	8	8.9
Do. rejecting Nos. (2) and (5),	5	8	8.3
Do. rejecting (2), (5) and (7),	5	8	7.0
Probable error of last result,			± 0.4
Probable error of a single one of the four best results,			± 0.8

The probability that this result is 10 sec. in error, is less than 0.0001. The probability that the error amounts to 25.0 sec. (the quantity required to include Lambert's longitude, reported to congress and accepted by that body) is too small to admit of computation. The high authority of Bowditch, Triesnecker, Wurm, De Ferrer and Paine, whose combined computations give the longitude of our prime meridian $5^h 8^m 7.0^s$ west of Greenwich, and the demonstrable error of Lambert's computations, which lead to a result of $5^h 7^m 42^s$, as reported to congress, leave to geographers no room for doubt as to the proper

* In the interval between the reading and printing of this paper, Robert Treat Paine, Esq., by means of three chronometers, carried by himself from Boston to Washington, and thence to Boston, through Philadelphia, obtained the following important results, for the longitude of the Capitol.

	<i>h.</i>	<i>m.</i>	<i>s.</i>
Washington—Boston, going,	23	49	96
“ “ returning,	23	50	06
Mean,	23	50	01
Washington—Philadelphia, going,	7	26	43
returning,	7	26	50
Mean,	7	26	46
Boston state house is, by Bowditch and Paine,	4	44	16.60
Philadelphia state house is, by my computations,	5	0	39.20
Whence, Washington by Boston,	5	8	6.61
Washington by Philadelphia,	5	8	5.66
Mean,	5	8	6.14
Adopted for the longitude of the Capitol,	5	8	7.0

location of the Capitol. It must, however, be generally regretted that the omission, on the part of Lambert, of an essential correction (which, for the eclipses of 1791 and 1811 had been previously pointed out by Bowditch), should, from force of circumstances, have exercised for many years so extensive an influence in the propagation of error.

